Electricity and Magnetism, Exam 4, 01/05/2017

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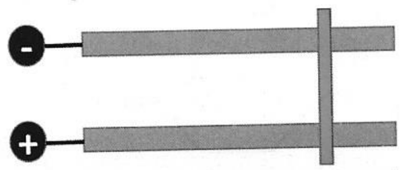
Exam reviewed by (name second examiner) Steven Hoekstra

12 questions; 5 pages

This is a multiple-choice exam. Write your name and student number on the answer sheet. Clearly mark the answer of your choice on the answer sheet. Only a single answer is correct for every question. The score will be corrected for guessing. Use of a (graphing) calculator is allowed. You may make use of the formula sheet (provided separately). The same notation is used as in the book, i.e. a bold-face **A** is a vector, T is a scalar.

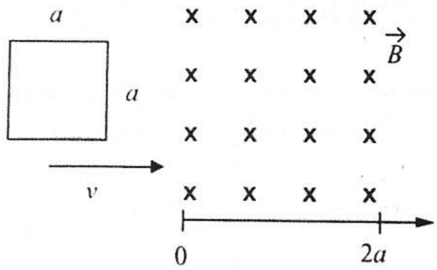
1. A pair of parallel conductors (rails) are firmly mounted on a horizontal surface. A conducting nonmagnetic bar orthogonal to the rails can freely slide along (but not across) the rails maintaining an electrical contact to both of them. The rails are connected to the power supply as shown in the figure. To which direction will the bar move?

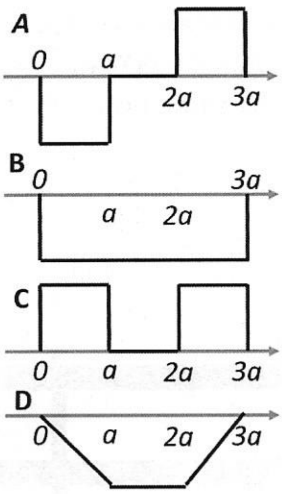


A. To the left
B. To the right
C. Depends on the exact material of the bar
D. The bar will not move

2. In the previous question, the force exerted upon the bar
A. Is proportional to the current
B. Is proportional to the square of the current
C. Depends on the exact material of the bar
D. There is no force

3. A square loop of side a moves with the constant speed V into a region in which a magnetic field of magnitude B exists perpendicular to the plane of the loop as shown in the figure. The ems induced in the loop as it enters, moves through, and exits the region of the magnetic field, is shown in the graphs below. Which graph is correct?





4. A long solenoid with radius a and n turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\boldsymbol{\phi}}$ direction. Find the electric field at a distance s from the axis inside the solenoid in quasistatic approximation.

Tip: The magnetic field inside the solenoid in the quasistatic approximation is $\mathbf{B} = \mu_0 n I\ \hat{\mathbf{z}}$

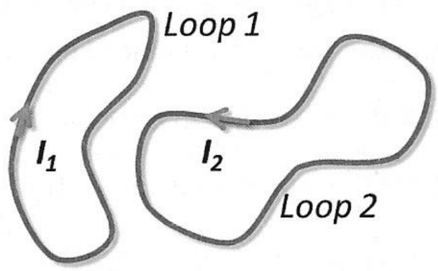
A. $\mathbf{E} = -\frac{\mu_0 n s}{2}\frac{dI(t)}{dt}\hat{\boldsymbol{\phi}}$

B. $\mathbf{E} = -\frac{\mu_0 n s}{2} I(t)\hat{\boldsymbol{\phi}}$

C. $\mathbf{E} = -\mu_0 n s\frac{dI(t)}{dt}\hat{\boldsymbol{\phi}}$

D. $\mathbf{E} = -\frac{\mu_0 n s}{2} I(t)\hat{\mathbf{z}}$

5. The mutual inductance M_{12} of the two loops depicted in the figure and currying the currents I_1 and I_2, depends on:



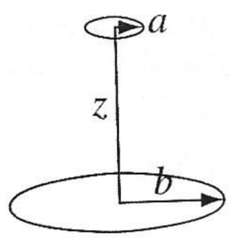
A. Current I_1

B. Current I_2

C. Currents I_1 and I_2

D. The sizes, shapes and relative positions of the loops

6. A small loop of wire (radius a) is held at a distance z above the center of a large loop (radius b), as shown in the figure. The planes of the two loops are parallel, and perpendicular to the common axis. The little loop is so small that you may consider the field of the big loop $\mathbf{B} = \frac{\mu_0 I_a}{2}\frac{b^2}{(b^2+a^2)^{\frac{3}{2}}}\hat{\mathbf{z}}$ to be essentially constant. The little loop is so small that you may treat it as a magnetic dipole with the field $\mathbf{B} = \frac{\mu_0 I_b b^2}{4}\left(2cos\theta\ \hat{\mathbf{r}} + sin\theta\ \widehat{\boldsymbol{\theta}}\right)$. Find the mutual inductance M of the loops. (Tip: recall properties of mutual inductance before running calculations!)



A. $M = \frac{\mu_0 \pi I_a I_b}{2}\frac{a^2 b^2}{(b^2+a^2)^{\frac{3}{2}}}$

B. $M = \frac{\mu_0 \pi I_a I_b}{2}\frac{a^2 b^2}{(b^2+a^2)^{\frac{1}{2}}}$

C. $M = \frac{\mu_0 \pi}{2}\frac{a^2 b^2}{(b^2+a^2)^{\frac{3}{2}}}$

D. $M = \frac{\mu_0 \pi}{2}\frac{a^2 b^2}{(b^2+a^2)^{\frac{1}{2}}}$

7. Find the emf induced in a square loop with sides of length a lying in the yz-plane in a region in which the magnetic field changes over time as $\mathbf{B}(t) = B_0 e^{-t/t_0}\hat{\boldsymbol{\imath}}$.

A. $emf = 0$

B. $emf = B_0 e^{-t/t_0}$

C. $emf = \frac{a^2 B_0}{t_0}$

D. $emf = \frac{a^2 B_0}{t_0} e^{-t/t_0}$

8. The magnetic field in a certain region is given by the expression $\mathbf{B}(t) = B_0 cos(kz - \omega t)\hat{\boldsymbol{j}}$. Find the curl of the induced electric field at that location.

A. $\nabla \times \mathbf{E} = -\omega B_0 sin(kz - \omega t)\hat{\boldsymbol{j}}$

B. $\nabla \times \mathbf{E} = -\omega B_0 cos(kz - \omega t)\hat{\boldsymbol{j}}$

C. $\nabla \times \mathbf{E} = -\left(\omega + k\frac{\partial z}{\partial t}\right)\omega B_0 sin(kz - \omega t)\hat{\boldsymbol{j}}$

D. $\nabla \cdot \mathbf{E} = -\omega B_0 sin(kz - \omega t)\hat{\boldsymbol{j}}$

9. As the current in an induction coil changes uniformly from 1 A to 6 A in 0.1 s, emf of -50 V is induced. What is inductance of the coil?

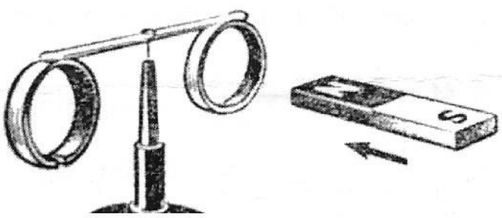
A. 1 H

B. 0.5 H

C. 1 T

D. 4 H

10. The magnet is pushed inside the metal *nonmagnetic* ring as shown in figure. What is the direction of the movement of the ring?



A. The ring does not move because it is nonmagnetic

B. It depends on the speed of the magnet

C. Toward the magnet

D. Opposite the magnet

11. Airbus 380 has a wingspan of 80 m and travels horizontally with a speed of 1080 km/h. Find emf induced between the tips of the wings. The vertical component of the Earth's magnetic field amounts to $5 \cdot 10^{-5}$ T.

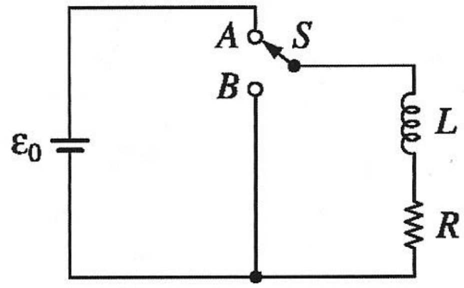
A. 0.6 A

B. 1.2 V

C. 0.6 V

D. 0

12. The circuit has been connected for a long time when suddenly, at time t = 0, switch S is quickly thrown from A to B, bypassing the battery. What is the current *I(t)* at any subsequent time *t*?



A. $I(t) = 0$ because the battery is disconnected

B. $I(t) = \frac{\varepsilon_0}{R}$

C. $I(t) = \frac{\varepsilon_0}{R} e^{-Rt/L}$

D. $I(t) = \frac{\varepsilon_0}{R}\left(1 - e^{-Rt/L}\right)$

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Erratum

Question 3. *"ems" should read "emf"*

Question 6. *Unfortunate mix-up between the indices of the two loops as well as z-direction has occurred. The correct formulation reads as*

6. A small loop of wire (radius *a*) is held at a distance z above the center of a large loop (radius *b*), as shown in the figure. The planes of the two loops are parallel, and perpendicular to the common axis. The little loop is so small that you may consider the field of the big loop $\mathbf{B} = \frac{\mu_0 I_b}{2}\frac{b^2}{(b^2+z^2)^{\frac{3}{2}}}\hat{\mathbf{z}}$ to be essentially constant. The little loop is so small that you may treat it as a magnetic dipole with the field $\mathbf{B} = \frac{\mu_0 I_a a^2}{4}\left(2cos\theta\ \hat{\mathbf{r}} + sin\theta\ \hat{\boldsymbol{\theta}}\right)$. Find the mutual inductance *M* of the loops. (Tip: recall properties of mutual inductance before running calculations!)

A. $M = \frac{\mu_0 \pi I_a I_b}{2}\frac{a^2 b^2}{(b^2+z^2)^{\frac{3}{2}}}$

B. $M = \frac{\mu_0 \pi I_a I_b}{2}\frac{a^2 b^2}{(b^2+z^2)^{\frac{1}{2}}}$

C. $M = \frac{\mu_0 \pi}{2}\frac{a^2 b^2}{(b^2+z^2)^{\frac{3}{2}}}$

D. $M = \frac{\mu_0 \pi}{2}\frac{a^2 b^2}{(b^2+z^2)^{\frac{1}{2}}}$